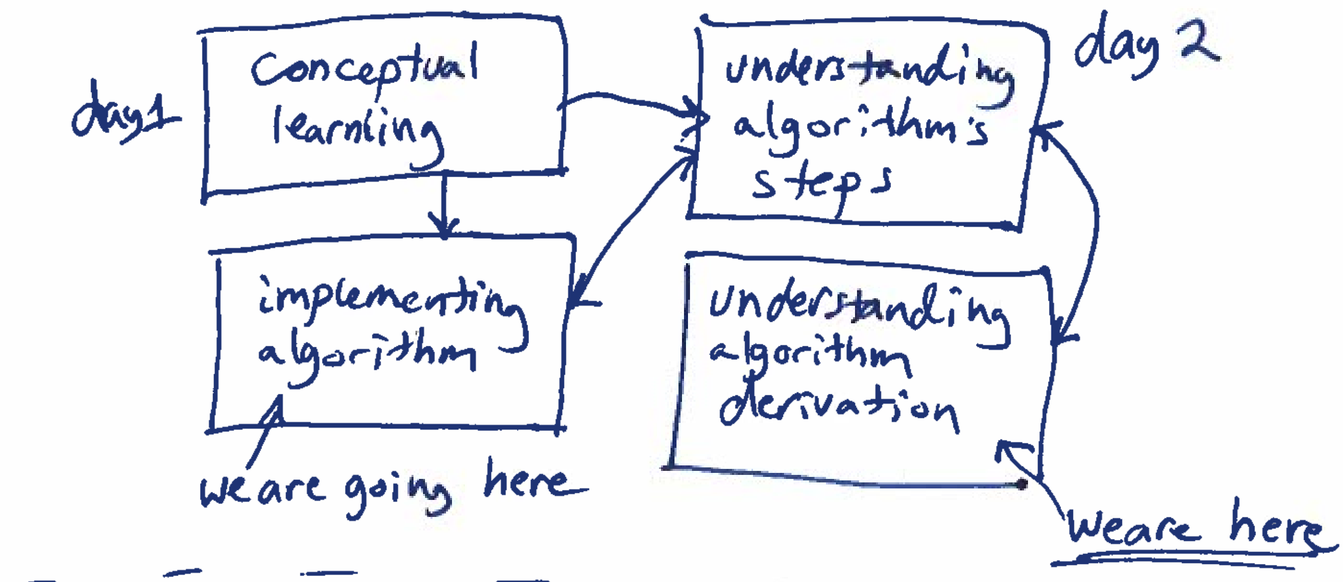
Algorithm learning process Map



Derivation of the Bayes filter for robot state estimation

(very general.... can be applied to any Markov process)

Before we start: why probabilities? why Bayes?

- probabilities: make underlying model of the world explicit... Express uncertainty and randomness.
- Bayes: a framework for how to ~~believe~~ reason about hidden causes given evidence

Tool 1 Bayes Rule

evidence given hypothesis

$$p(H \mid E) = \frac{p(E \mid H)\,p(H)}{p(E)}$$

Tool 2 Product Rule

$$p(A,B) = p(A)\,p(B \mid A)$$

(you choose the order)

Tool 3 Conditional Independence

$$p(A,B \mid C) = p(A \mid C)\,p(B \mid C)$$

is true when A and B are conditionally independent given C.

When is this true? We have some formal tools, but we're not going to go into it.

Tool 4 Law of total probability

Given some mutually exclusive and exhaustive set of events $B_1 \ldots B_n$

$$p(A) = p(A,B_1) + p(A,B_2) + \ldots + p(A,B_n)$$

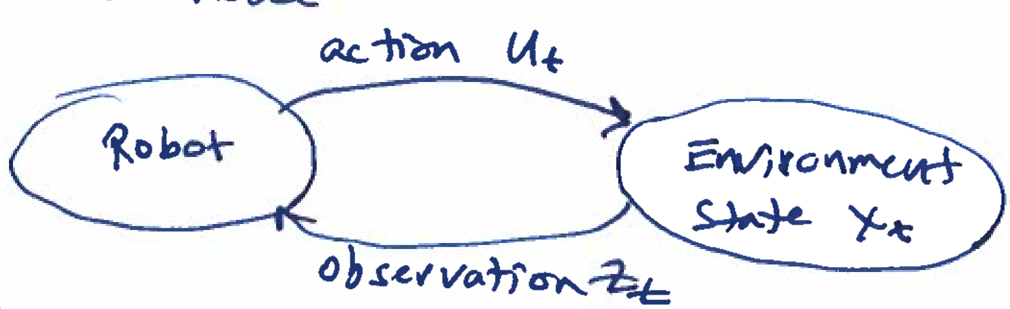
Invite folks to leave if they want an independent experience

Bayes Filter via Robot State Estimation

Strategy: derive for particular problem and then state the general form.

(see "Probabilistic Robotics" for detail)

Notation and Model



Time line:

robot starts in state	x_0
does action	u_1
arrives in state	x_1
observes	z_1
does action	u_2
arrives in state	x_2
observes	z_2

time ↓

Goal:

compute $\boxed{p(X_t \mid u_1 \dots u_t, z_1 \dots z_t)}$

Ex

$$x_t \triangleq \begin{cases} 1 & \text{door open @ time } t \\ 0 & \text{" closed @ " "} \end{cases}$$

$$u_t \triangleq \begin{cases} 1 & \text{robot pushes door @ time } t \\ 0 & \text{" doesn't push " " "} \end{cases}$$

$$z_t \triangleq \begin{cases} 1 & \text{robot senses door open @ time } t \\ 0 & \text{" doesn't sense " " " "} \end{cases}$$

Let's model our system!

$\boxed{\text{initial state model}}$ $p(X_0=1) = 1/2 \quad (\Rightarrow p(X_0=0)=1/2)$

$\boxed{\text{motor model}}$

$$p(x_t=1 \mid x_{t-1}=0, u_t=0) = 0, \quad p(x_t=1 \mid x_{t-1}=0, u_t=1) = 4/5$$
$$p(x_t=1 \mid x_{t-1}=1, u_t=0) = 1, \quad p(x_t=1 \mid x_{t-1}=1, u_t=1) = 1$$

$\boxed{\text{Sensor Model}}$ $p(z_t=1 \mid x_t=1) = 3/5, \quad p(z_t=1 \mid x_t=0) = 1/5$

Problem 1

$$p(x_1=1 \mid z_1=1, u_1=0)?$$

$$p(x_1=1 \mid z_1=1, u_1=0) = \frac{p(z_1=1 \mid x_1=1, u_1=0)\, p(x_1=1 \mid u_1=0)}{p(z_1=1 \mid u_1=0)}$$ Bayes' Rule

$3/5 \rightarrow$

$$\propto p(z_1=1 \mid x_1=1, \cancel{u_1=0})\, p(x_1=1 \mid u_1=0)$$

$$p(x_1=1 \mid u_1=0) = p(x_1=1, x_0=0 \mid u_1=0) + p(x_1=1, x_0=1 \mid u_1=0)$$ $\leftarrow$ total prob

$$= p(x_0=0 \mid \cancel{u_1=0})\, p(x_1=1 \mid x_0=0, u_1=0) + p(x_0=1 \mid \cancel{u_1=0})\, p(x_1=1 \mid x_0=1, u_1=0)$$

$$= (1/2)(0) + (1/2)(1) = (1/2)$$

$$\Rightarrow p(x_1=1 \mid z_1=1, u_1=0) \propto (1/2)(3/5) = 3/10$$

Next we need: $p(x_1=0 \mid z_1=1, u_1=0)$

From before: (analogous)

$$p(x_1=0 \mid z_1=1, u_1=0) \propto \underbrace{\cancel{p(z_1=1 \mid x_1=0)}}_{1/5}\, p(x_1=0 \mid u_1=0)$$

and... $p(x_1=0 \mid u_1=0) =$

$$\underbrace{\cancel{p(x_0=0)}}_{1/2}\,\underbrace{\cancel{p(x_1=0 \mid x_0=0, u_1=0)}}_{1} + \underbrace{\cancel{p(x_0=1)}}_{1/2}\,\underbrace{\cancel{p(x_1=0 \mid x_0=1, u_1=0)}}_{0}$$

$$= 1/2$$

$\therefore\ p(x_1=0 \mid z_1=1, u_1=0) \propto (1/5)(1/2) = (1/10)$

$$\Rightarrow\ p(x_1=1 \mid z_1=1, u_1=0) = \frac{3/10}{1/10 + 3/10} = 3/4$$

Problem 2

$p(x_2=1 \mid z_1=1, z_2=1, u_1=0, u_2=1)$?

Bayes' Rule

$$= \frac{p(z_2=1 \mid z_1=1, x_2=1, u_1=0, u_2=1)\, p(x_2=1 \mid z_1=1, u_1=0, u_2=1)}{p(z_2=1 \mid z_1=1, u_1=0, u_2=1)}$$

$$\propto \underbrace{p(z_2=1 \mid \cancel{z_1=1}, x_2=1, \cancel{u_1=0}, \cancel{u_2=1})}_{3/5}\, p(x_2=1 \mid z_1=1, u_1=0, u_2=1)$$

$$p(x_2=1 \mid z_1=1, u_1=0, u_2=1) = p(x_1=0, x_2=1 \mid z_1=1, u_1=0, u_2=1) + p(x_1=1, x_2=1 \mid z_1=1, u_1=0, u_2=1)$$

law of Total prob.

$$= p(x_1=0 \mid z_1=1, u_1=0, \cancel{u_2=1})\, p(x_2=1 \mid x_1=0, \cancel{z_1=1}, \cancel{u_1=0}, u_2=1)$$
$$+ p(x_1=1 \mid z_1=1, u_1=0, \cancel{u_2=1})\, p(x_2=1 \mid x_1=1, \cancel{z_1=1}, \cancel{u_1=0}, u_2=1)$$

$$= (1/4)(4/5) + (3/4)(1) = 19/20$$

where did this come from?

$$\Rightarrow p(x_2=1 \mid z_1=1, z_2=1, u_1=0, u_2=1) \propto 57/100$$

Analogously

$$p(x_2=0 \mid z_1=1, z_2=1, u_1=0, u_2=1)$$

$$\propto \cancel{p(z_2=1 \mid x_2=0)}\ 1/5 \quad p(x_2=0 \mid z_1=1, u_1=0, u_2=1)$$

and ...

$$p(x_2=0 \mid z_1=1, u_1=0, u_2=1) =$$

$$p(x_1=0 \mid z_1=1, u_1=0)\, p(x_2=0 \mid x_1=0, u_2=1)$$

$$+\ p(x_1=1 \mid z_1=1, u_1=0)\, p(x_2=0 \mid x_1=1, u_2=1)$$

$$= (1/4)(1/5) + (3/4)(0) = 1/20$$

$$\Rightarrow \quad p(x_2=1 \mid z_1=1, z_2=1, u_1=0, u_2=1) \propto (1/5)(1/20) = 1/100$$

$$p(x_2=1 \mid z_1=1, z_2=1, u_1=0, u_2=1) = \frac{57/100}{57/100 + 1/100}$$

$$= 57/58$$

Bayes' Binary

$$p(x_t=1 \mid u_1 \ldots u_t, z_1 \ldots z_t) \propto p(z_t \mid x_t=1) \times$$

$$\Big[p(x_t=0 \mid u_1 \ldots u_{t-1}, z_1 \ldots z_{t-1})\, p(x_t=1 \mid x_{t-1}=0, u_t)$$

$$+\ p(x_{t-1}=1 \mid u_1 \ldots u_{t-1}, z_1 \ldots z_{t-1})\, p(x_t=1 \mid x_{t-1}=1, u_t) \Big]$$

Bayes' discrete

$$p(x_t=i \mid u_1 \ldots u_t, z_1 \ldots z_t) \propto p(z_t \mid x_t=i) \sum_{j=1}^{K} p(x_{t-1}=j \mid u_1 \ldots u_{t-1}, z_1 \ldots z_{t-1})\, p(x_t=i \mid x_{t-1}=j, u_t)$$

How does this scale?